Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Differential Geometry
Subject Code: 5SC02DIG1 **Branch: M.Sc. (Mathematics)**
Semester: 2 **Date:02/05/2017** **Time:02:00 To 05:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Convert parametrized curve $\bar{r} : R \to R^2$ by $\bar{r}(t) = (a\cos t\,,\ b\sin t)$ in to Cartesian curve where $a, b \in R - \{0\}$. **(01)**

b. Define: Planar curve. **(01)**

c. Check whether the curve $\bar{r}(t) = (\cos t, \sin t, 3\sin t + 4\cos t)$ is planar or not? **(01)**

d. Define: Tangent of curve **(01)**

e. Define: Signed curvature. **(01)**

f. Consider the logarithmic spiral $\bar{r}(t) = (e^t \cos t\,, e^t sint)$ then prove that the angle between $\bar{r}(t)$ and $\dot{\bar{r}}(t)$ is independent of t. **(02)**

Q-2 **Attempt all questions** **(14)**

a) If $\bar{r}$ is a regular curve in R^3 then prove that the curvature $k = \dfrac{\|\ddot{\bar{r}} \times \dot{\bar{r}}\|}{\|\dot{\bar{r}}\|^3}$ **(06)**

b) Compute curvature and torsion of $\bar{r}(t) = (a\cos t,\ a\sin t, bt)$. **(06)**

c) Let S and $\tilde{S}$ be smooth surfaces. Let $f : S \to \tilde{S}$ be a smooth map and let $p \in S$.In usual notation prove that the derivative $D_p f : T_p S \to T_{f(p)}\tilde{S}$ is a linear map. **(02)**

OR

Q-2 **Attempt all questions** **(14)**

a) Let $\bar{r} : (a, b) \to R^3$ be a regular curve with nowhere vanishing curvature. Then prove that the torsion τ of $\bar{r}$ is $\dfrac{(\dot{\bar{r}} \times \ddot{\bar{r}}) \cdot \dddot{\bar{r}}}{\|\dot{\bar{r}} \times \ddot{\bar{r}}\|^2}$ **(06)**

b) Prove that a re-parametrization of a regular curve is regular. **(04)**

c) Show that the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is a convex curve. **(04)**

Q-3 **Attempt all questions** **(14)**

a) Let $\bar{r} : (a, b) \to R^2$ be a unit speed curve and let $s_0 \in (a, b)$. Let $\phi_0 \in R$ be such that $\dot{\bar{r}}(s_0) = (\cos\phi_0\,, \sin\phi_0)$. Then prove that there exists a unique smooth **(07)**

map $\phi: (a,b) \to R$ such that $\dot{\bar{r}}(s) = (\cos\phi(s), \sin\phi(s))$ for all $s \in (a,b)$ and $\phi(s_0) = \phi_0$.

b) If $K: (a,b) \to R$ is a smooth map then prove that there is a unit speed curve $\bar{r}: (a,b) \to R^2$ whose signed curvature is K. Also if $\bar{r}: (a,b) \to R^2$ is unit speed curve with signed curvature is K then prove that there is a direct isometry M of R^2 such that $\tilde{r} = M \circ \bar{r}$. **(07)**

OR

Q-3 a) Let $F: [0,\pi] \to R$ be a smooth map satisfying $F(0) = F(\pi) = 0$ then prove that $\int_0^\pi \dot{F}^2\, dt \geq \int_0^\pi F^2\, dt$. **(05)**

b) Compute first fundamental form of the surface $\sigma(u,v) = (f(u)\cos v, f(u)\sin v, g(u))$. **(05)**

c) Check whether the surface $S = \{(x,y,z) \in R^3 \mid z = x^2 + y^2\}$ is smooth. **(04)**

SECTION – II

Q-4 Attempt the Following questions **(07)**

a. Define: Surface. **(01)**
b. Define: Oriented Surface. **(01)**
c. Define: Unit normal to surface at a point. **(01)**
d. Define: Local isometry. **(01)**
e. Define: Umbilical point. **(01)**
f. Define: Geodesic **(01)**
g. State Bonnet's theorem. **(01)**

Q-5 Attempt all questions **(14)**

a) Calculate second fundamental form of sphere. **(07)**

b) Compute the Gaussian curvature and the mean curvature of the surface $z = f(x,y)$ where f is smooth map. **(07)**

OR

Q-5 a) Compute the principal curvatures on the surface $\sigma(u,v) = (u, v, uv)$. **(07)**

b) Prove that surface area of sphere having radius r is $4\pi r^2$. **(04)**

c) Find the image of the Gauss map for $\sigma(u,v) = (u, v, u^2 + v^2)$ where $u, v \in R$. **(03)**

Q-6 Attempt all questions **(14)**

a) State and prove Euler's theorem. **(05)**

b) State and prove Meunier's theorem. **(05)**

c) Prove that a unit speed curve on a surface S is a geodesic if and only if its geodesic curvature is zero everywhere. **(04)**

OR

Q-6 a) Determine the parabolic, elliptic and hyperbolic points on the surface $\sigma(u,v) = ((b + a\cos v)\cos u,\ (b + a\cos v)\sin u, a\sin v)$, where $b > a$. **(09)**

b) Compute Christoffel's symbols of second kind for the circular cylinder **(05)**

$$\sigma(u,v) = (\cos u, \sin u, v).$$